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The double integral method applied to separation calculation

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Abstract—Using a specially chosen velocity profile, the double integral method is used to derive an easily applied analytical result for the location of the laminar separation point. Predictions from this result are compared to the accepted separation point locations for many different adverse pressure gradients with very good agreement. An effective velocity profile across the boundary layer was developed and was found to compare well to an exact solution and to experimental data in separated and near separated flows. It also provides an explanation for the better performance of the double integral method compared to the standard integral method.

INTRODUCTION

The basic idea for the double integral method was put forth by Volkov [1]. Since then it has been applied to many different types of problems, namely, conduction in solids by Volkov and Li Orlov [2], and by Chung and Yeh [3], prediction of skin friction and heat transfer with transpiration, Zien, [4], [5], transient forced convection and aerodynamic heating, Sucec [6], and to phase change problems, El-Genk and Cronenberg [7].

In the present work, the focus is on the application of the double integral method to the prediction of the location of the laminar separation point and the velocity profile across a boundary layer flow.

The Von Karman type, single parameter, integral method gave the first practical procedure for calculation of laminar boundary layers, but gives poor predictions when used on the problem of separation. As Curle, [8], points out, the Karman–Pohlhausen procedure is 29% in error for the separation location for the linearly retarded flow when a polynomial using terms up through y^4 is used as the velocity profile. A recent integral method by Thomas and Amminger, [9], has an error of 10% in the vicinity of separation when using their recommended approximation for adverse pressure gradients. The more complicated multiparameter integral method of Abbott and Bethel, [10], yields results which compare well to computer solution calculations of separation location. Separation predictions by series solution, and the associated problems, are typified by the work of Howarth [11] while Smith and Clutter, [12], provide finite difference computer solutions for separation prediction. Curle and Skan [13], take the exact solution form for two asymptotically large unfavorable pressure gradients and then determine the optimum value of a constant in the solution so as to minimize the difference between the separation point predicted by this form and the actual value of the separation point for a number of different free stream velocities, $u_s(x)$, which lead to separation. The final result of this "fitting" procedure is what is usually referred to as Stratford's result (Curle [8]).

The present work uses a velocity profile, specifically developed for the separation point, in the double integral method to easily and accurately predict separation location for arbitrary, specified $u_s(x)$.

ANALYSIS

Steady, laminar, constant property, planar two dimensional (2D) boundary layer flow with free stream velocity variation, $u_s(x)$, will be considered here. The x momentum theorem is applied to a control volume dx long by y high. Using ξ as a dummy variable for y, the result is as follows.

$$\nu\left(\frac{\partial u}{\partial \xi}\right)_{\xi=0} = \nu\frac{\partial u}{\partial y} + u_s\frac{du_s}{dx}y + u\frac{\partial}{\partial x}\int_0^y u\,d\xi - \frac{\partial}{\partial x}\int_0^y u^2\,d\xi. \quad (1)$$

Now, as

$$y \to \delta, \quad \frac{\partial u}{\partial y} \to 0, \quad u(x,y) \to u_s(x)$$

and equation (1) becomes,

$$\nu\left(\frac{\partial u}{\partial \xi}\right)_{\xi=0} = u_s\frac{du_s}{dx}\delta + u_s\frac{d}{dx}\int_0^\delta u\,d\xi - \frac{d}{dx}\int_0^\delta u^2\,d\xi. \quad (2)$$

Equation (2), is seen to be the von Karman x momentum integral equation.

NOMENCLATURE

a_j	Profile parameters defined by equation (4)
C	constant used in some velocity profiles
$C_f = 2\tau_w/\rho u_s^2$	local skin friction coefficient
g	constant defined in equation (7)
j	an index
L	reference length
m, n	constants used as a power in some velocity profiles
N	number of terms used in an approximating sequence
Re_L	$u_0 L/\nu$, a Reynolds number
Re_x	$u_s x/\nu$ local length Reynolds number
u, u_0, u_s	local x component, maximum or reference, and freestream velocities, respectively
x	space coordinate along the body
X	x/L, nondimensional x coordinate
y	space coordinate perpendicular to body surface
Y	y/L, nondimensional y coordinate.

Greek symbols

δ	local boundary layer thickness
θ	local momentum thickness
σ	dummy variable
ν	kinematic viscosity
ξ	dummy variable
ρ	mass density
τ_w	local wall shear stress.

The usual single parameter integral approach is to insert an approximating sequence for $u(x, \xi)$ into equation (2) and solve for $\delta(x)$. Note that in doing this, the approximating sequence must be directly differentiated with respect to ξ in the term on the left side of equation (2). Conceivably, the direct derivative of an approximating sequence could have a much greater error than exhibited by the sequence itself, so the suggestion given by Volkov [1] will be used. Thus, equation (2) will be used to find the needed derivative on the left by evaluating the terms on the right once $\delta(x)$ has been found. The parameter, $\delta(x)$, is found from a separate equation, the modified or double integral equation, arrived at by integrating equation (1) with respect to y across the boundary layer thickness. The increase in accuracy of predicted results, when using this technique instead of the usual integral method, can be seen for a variety of flow and convective heat transfer problems in the works of Zien [4], [5] and Sucec [6]. The procedure just described gives the double integral equation shown next.

$$\left[\frac{u_s\delta}{2}\frac{du_s}{dx}+u_s\frac{d}{dx}\int_0^\delta u\,d\xi-\frac{d}{dx}\int_0^\delta u^2\,d\xi\right]\delta = \nu u_s+\int_0^\delta\left(u\frac{\partial}{\partial x}\int_0^y u\,d\xi\right)dy - \int_0^\delta\left(\frac{\partial}{\partial x}\int_0^y u^2\,d\xi\right)dy. \quad (3)$$

During the construction and testing of a polynomial approximating sequence for the velocity, $u(x, y)$, to be used to solve equation (3) for $\delta(x)$, it was found that a critical requirement, for very accurate separation prediction, was that $(\partial u/\partial y)_{y=0} = 0$ for all x. This condition is needed to force the approximate velocity profile as close to a separation profile as possible. The use of the double integral method allows this latitude because the actual separation condition, that $\partial u/\partial y = 0$ at $y = 0$ and at x_s, is formed from equation (2), not from a direct differentiation of the sequence for u.

The approximating sequence is shown next.

$$u(x, y) = \sum_{j=1}^{N} a_j(x) y^j. \quad (4)$$

The coefficient functions $a_j(x)$ are found in terms of $\delta(x)$ by use of this $\partial u/\partial y$ requirement, boundary conditions and other conditions derivable from the partial differential x momentum equation at $y = 0$ and $y = \delta$.

$$y = 0 \quad u, \frac{\partial u}{\partial y} \text{ and } \frac{\partial^3 u}{\partial y^3} = 0 \quad (5)$$

$$y = \delta \quad u = u_s(x) \quad \frac{\partial u}{\partial y} = \frac{\partial^2 u}{\partial y^2} = 0.$$

Applying the conditions, equation (5), to the profile, equation (4), yields the following one parameter, $\delta(x)$, approximating sequence.

$$\frac{u(x, y)}{u_s(x)} = \frac{10}{3}\frac{y^2}{\delta^2}\left[1-\frac{3}{2}\frac{y^2}{\delta^2}+\frac{4}{5}\frac{y^3}{\delta^3}\right]. \quad (6)$$

It is to be noted that equation (6) was not forced to satisfy the condition that $\partial^2 u/\partial y^2 = -(1/\nu)u_s du_s/dx$ at $y = 0$, a compatibility condition that is satisfied by virtually all approximate velocity profiles used in single parameter integral methods for separation prediction. However, this condition complicates the solution for $\delta(x)$ in that a finite difference solution of the differential equation becomes necessary. Since the work being presented here involves the development of a simple, easily used method, the second derivative condition at $y = 0$ was purposely avoided to allow an analytical solution for $\delta(x)$. This is another example of the additional flexibility given by the double integral method, along with the previously mentioned ability to set $\partial u/\partial y = 0$ at $y = 0$ for all x and still be able to solve for the actual location of separation, at x_s.

Substitution of the approximate velocity profile, equation (6), into equation (3) and defining $z = \delta^2$ gives a linear first order differential equation for z.

Solving by the integrating factor method yields the general solution for $\delta(x)$.

$$\delta(x) = \left[\frac{8316\nu}{301 u_s^g(x)}\right]^{1/2}\left[\int_o^x u_s^{g-1}(\sigma)\,d\sigma\right]^{1/2} \quad g = \frac{6076}{903}. \tag{7}$$

At the separation point $(\partial u/\partial\xi)_{\xi=0} = 0$. Using this on the left-hand side of equation (2), inserting equation (6) and (7) into the right-hand side and performing the operations gives the following general separation condition.

$$\frac{-44891}{9030}\frac{du_s}{dx} = \frac{u_s^g(x)}{\int_o^x u_s^{g-1}(\sigma)\,d\sigma} \quad \text{at } x = x_s. \tag{8}$$

This simple laminar separation relation, equation (8), is one of the main results of the present work.

Effective velocity profile for the method

The fact that the double integral method, which is being used here, does not directly differentiate the approximating sequence for the velocity allows the use of very simple approximate profiles while at the same time yielding the boundary derivative very accurately (see Zien [4], [5] and Sucec [6].) However, the method also seems to define an effective, underlying higher order velocity profile, better than equation (6), which it is using. Consider equation (1) and solve it for $\partial u/\partial y$, giving:

$$\frac{\partial u}{\partial y} = \left(\frac{\partial u}{\partial \xi}\right)_{\xi=0} - \frac{u_s}{\nu}\frac{du_s}{dx}y - \frac{u}{\nu}\frac{\partial}{\partial x}\int_o^y u\,d\xi + \frac{1}{\nu}\frac{\partial}{\partial x}\int_o^y u^2\,d\xi. \tag{9}$$

Next, integrate equation (9) with respect to y, using equation (6) for u in the integrals and finding $(\partial u/\partial\xi)_{\xi=0}$ from equation (2). When all the integrations and other operations are performed, one arrives at the following expression for u after replacing y/δ by η

$$\frac{u(x,\eta)}{u_s(x)} = \left[\frac{52}{77}\frac{\delta^2}{\nu}\frac{du_s}{dx} + \frac{80}{693}\frac{u_s\delta}{\nu}\frac{d\delta}{dx}\right]\eta + \frac{\delta^2}{\nu}\frac{du_s}{dx}\left[\frac{-\eta^2}{2} + \frac{10}{81}\eta^6 - \frac{10}{126}\eta^8 + \frac{\eta^{10}}{18} - \frac{4}{99}\eta^{11} + \frac{8}{891}\eta^{12}\right] + \frac{u_s\delta}{\nu}\frac{d\delta}{dx}\left[-\frac{20}{81}\eta^6 + \frac{130}{252}\eta^8 - \frac{20}{81}\eta^9 - \frac{2}{9}\eta^{10} + \frac{20}{99}\eta^{11} - \frac{40}{891}\eta^{12}\right]. \tag{10}$$

It is seen that equation (10) is the velocity profile which, when directly differentiated with respect to y gives the same result, namely equation (9), as is yielded by the double integral method when it employs the much simpler velocity profile, equation (6). As is evident, equation (10) contains powers of y from 1 to 12 and also has the parameter $(\delta^2/\nu)du_s/dx$ in it. Actually, it can be shown that the velocity profile given by equation (10) satisfies the compatibility condition at the wall mentioned earlier, namely that $\partial^2 u/\partial y^2 = -(u_s/\nu)\,du_s/dx$ at $y = 0$. On the other hand, the original velocity profile, equation (6), is a lower order profile than is equation (10) and does not satisfy the conditions just mentioned. Equation (6) was used in the operations that yield equation (10). Thus, it is argued that equation (10), not equation (6), is the underlying, effective velocity profile used by the method since the differentiation of equation (10) gives the derivative, $\partial u/\partial y$, that is actually used by the double integral method. This also suggests that the reason for the double integral method performing so much better than the standard integral method, as is demonstrated in Zien, [4], [5] and Sucec [6], when both methods use the same approximating sequence, is due to the velocity profile being the effective profile just discussed rather than the original lower order sequence. This is perhaps most easily seen when very low order approximating sequences are employed, such as the linear profiles used in Zien, [4], [5]. However, these simple linear profiles in the double integral method lead to higher order effective profiles (as equation (6) leads to the effective equation (10) in the present work) which should be used to portray the variation of the dependent variables across the boundary layer.

To illustrate that the higher order effective velocity profile of the double integral method, (equation (10) of this work) rather than the simple approximating sequence (equation (6) of this work), should be used if one wishes to predict the detailed velocity distribution across the boundary layer, we consider the flow near a stagnation point of a cylinder. Here, $u_s(x) = CX$ is inserted into equation (7) to solve for δ. With this, the effective velocity profile is given by equation (10), while the simpler profile used to develop the expression for $\delta(x)$ is equation (6). In Fig. 1, $u/u_s(x)$ is plotted vs y/δ as a solid line for equation (10) and as a dashed curve for equation (6). Also plotted there is the exact similarity solution from White [14]. The effective "edge" of the boundary layer was taken at $u/u_s(x) = 0.99$ which occurs when the similarity variable used in [14] has a value of 2.4. It is seen that the effective velocity profile given by equation (10) represents the exact velocity profile very well throughout most of the boundary layer, while equation (6) is an extremely poor representation of the true velocity profile. Certainly, the error involved in using equation (6) is not unexpected since it was deliberately chosen to be a separation profile and it is being used here far upstream of the separation point on the cylinder. Yet,

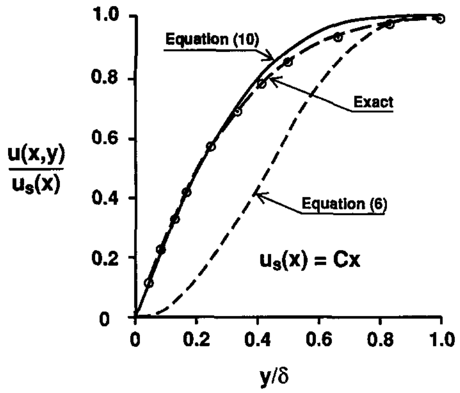

Fig. 1. Comparison of predicted velocity profiles from equation (10) and from equation (6) to the exact result for $u_s(x) = CX$.

at the same time, the underlying effective velocity profile implied by the double integral method, equation (10), does a good job of predicting the true velocity profile even here at the stagnation point. However, right at the separation point equations (10) and (6) give virtually the same velocity profile.

RESULTS AND DISCUSSION

Table 1 compares the separation point locations predicted by the present method to the accepted or "exact" values for sixteen different free stream velocities, $u_s(x)$. These accepted or exact values are mostly from finite difference solutions of the boundary layer equations or from methods supported by finite difference results. This table includes the classic free stream velocity variations which lead to separation such as the linear retarded flow, case 1, the flows of Curle, cases 3–5, the Tani flows 12–14 and the Gortler flows, 6–10. The exact values of separation location are taken, for the most part, from White [14]. However, cases 3 and 5 come from Curle [8], the Hiemenz flow, case 11, is from Smith and Clutter [12] and case 15 is from Abbott and Bethel [10].

The predictions of the present double integral method shown in Table 1 seem very satisfactory. The predicted values of separation location, the first fifteen cases, have an average error magnitude of 1.5% and an RMS error of 1.9%. Next, a couple of the cases of special interest or note will be discussed in more detail.

Case 1 of Table 1, the linearly retarded flow, is a classic test case for separation prediction techniques. Equation (8) of the present method predicts an x_s value which is less than 0.4% lower than the exact value. For this case, Howarth, [11], has used his series technique to predict the velocity distribution across the layer at the separation point. The curves at the right in Fig. 2 compare Howarth's predictions, the dashed curve, to the predicted effective velocity profile of the present work, namely, equation (10) plotted as the solid curve. Agreement between the two predictions is seen to be very good.

The last case in Table 1, the similarity separating flow, is an interesting and challenging one. This power law free stream velocity profile causes incipient separation for all x when the power $m = -0.0904$, Curle [8]. The well known method associated with Stratford fails to yield any solution at all for a value of m which causes separation. When Abbott and Bethel [10] used their multiparameter integral method on the strongly retarded power law flow with a value of $m = -0.0876$, not quite a separating similarity flow, they were unable to calculate shear stress, using the order of the approximation deemed necessary, because of excessive computer time. Presumably this indicates that their method may not be able to predict the value of m needed to cause separation. Surprisingly, even the

Table 1. Comparison of predictions of present work with the "exact" separation point locations for different free stream velocity variations

Case	$u_s(X)/u_o$	X at separation "Exact"	Present work	Error* [%]
1	$1-X$	0.119863	0.119445	0.35
2	$\sin X$	1.823	1.80411	1.04
3	$X-X^3+0.07885X^5$	0.6647	0.6678	0.47
4	$X-X^3$	0.655	0.6499	0.78
5	$X-X^3-0.12156X^5$	0.6245	0.6277	0.51
6	$(1-X)^{1/2}$	0.21819	0.21550	1.23
7	$(1-X)^2$	0.06368	0.06312	0.88
8	$(1+X)^{-1}$	0.15111	0.15184	0.48
9	$(1+X)^{-2}$	0.07130	0.07114	0.22
10	$\cos X$	0.38881	0.3786	2.62
11	$1.814X-0.271X^3$ $-0.0471X^5$	80°	78.8°	1.5
12	$1-X^2$	0.27215	0.26489	2.67
13	$1-X^4$	0.46271	0.4471	3.37
14	$1-X^8$	0.64122	0.6204	3.24
15	$1-X^3$	0.38234	0.3701	3.2
16	CX^m	$-0.0904 = m$	$-0.09346 = m$	3.4

*Absolute value of error.

improved version of Thwaites method, Curle and Skan [13], which uses an underlying velocity profile based partly on similarity solutions, has some difficulty with this case of similarity separating flow. It predicts $m = -0.10$, for separation, a result which is 10.6% in error. The present double integral method predicts separation at $m = -0.0935$, or an error of 3.4%

Another separation case was studied which is not shown in Table 1. Schubauer, [15], experimentally detected flow separation at $X = 1.99 \pm 0.02$ on an elliptic cylinder. However, when equation (8) was used with the measured free stream velocity variation, no separation was predicted. The calculations did indicate that the boundary layer was close to separation at the measured location, the same conclusion reached in the finite difference solution for this case by Smith and Clutter [12]. Schubauer [15] had also measured the velocity profile across the boundary layer at $X = 1.946$, just upstream of separation. These experimental data points are shown as circles on the left side of Fig. 2. The solid curve represents the effective velocity profile prediction, equation (10), of the present double integral method. It is seen that the predicted profile agrees well with the data for this near separation condition. If equation (6) were used to represent the velocity profile at this near separation condition, it is found that the velocity at all six data points closest to the wall is underpredicted as a consequence of the zero velocity gradient at the wall given by this separation profile, equation (6). Though the present case is less severe, this difference between the effective velocity profile, equation (10), and the original approximate velocity profile, equation (6), was shown earlier in Fig. 1 and the associated discussion.

Comparison to common separation prediction methods

In comparing average or RMS % error of the present method to the other easily applied methods, namely those of Thwaites and of Stratford, it would be well to remember that the original form of these two methods has undergone an adjustment or a fitting process by Curle and Skan [13] to enable the predictions to more nearly match, on the average, the accepted exact values of separation location. The adjustments made by Curle and Skan [13] to the method of Thwaites reduce the magnitude of its average error from 4% in the original method to 2.4% for the ten cases used there. In changing the value of a constant originally recommended by Stratford, for general application of his method, as 0.0076 (Curle [8]) to the currently used value of 0.0104, Curle and Skan [13] reduce the average magnitude of the error from 7.9 to 1.2% for the nine cases they considered. The present double integral method has an average magnitude error of 1.6% for these same cases without any adjustment or tuning of the constants in equation (8). Additionally, the present method gives velocity profiles at and very near separation.

Lastly, the question of using the present method to

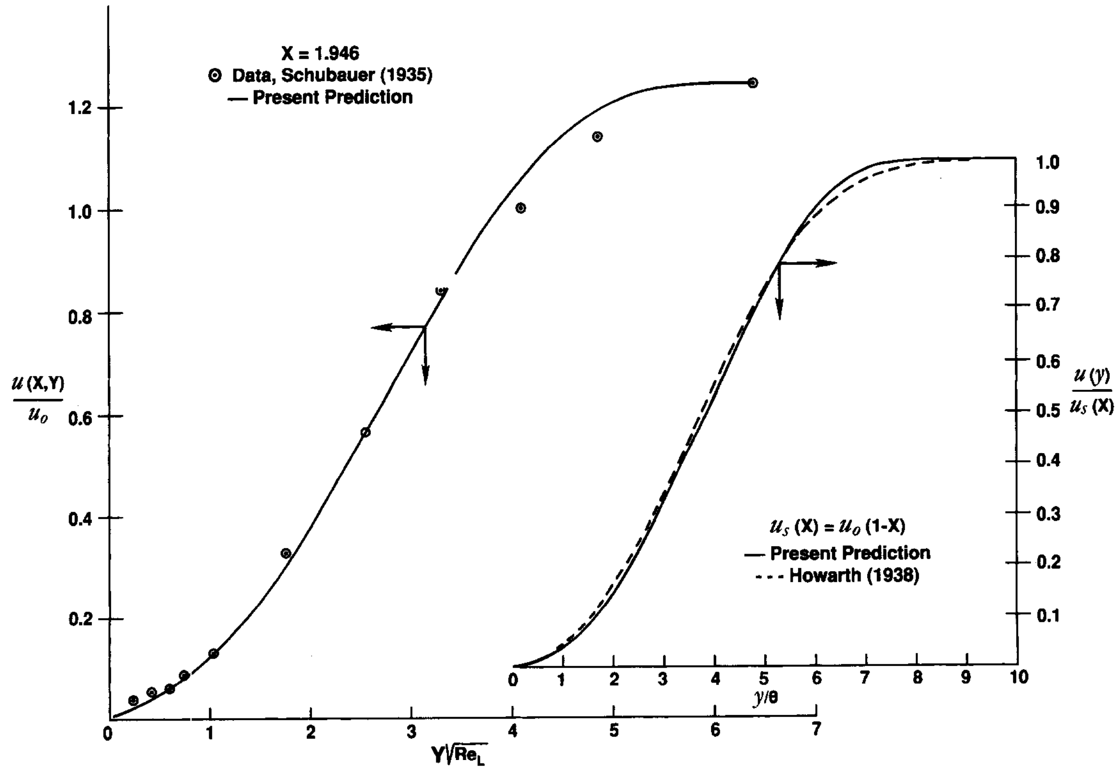

Fig. 2. Comparison of predicted boundary layer velocity profiles, from equation (10), to Schubauer's [15] data and to Howarth's [11] result.

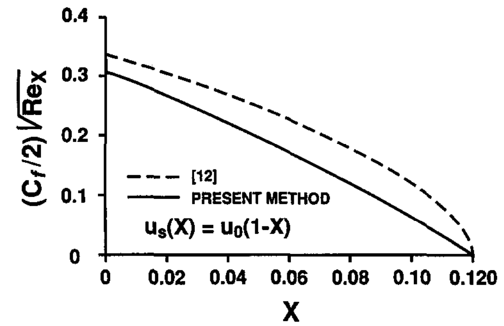

Fig. 3. Predicted skin friction compared to finite difference predictions for $u_s(X) = u_o(1-X)$.

predict skin friction along the surface, before separation is reached, will be discussed. It is well known that the double integral method, using very low order approximating sequences, can accurately give skin friction away from the separation point, Zien, [4], [16]. However, this is not recommended for the present work which should be used only to predict the separation location and details of the velocity distribution across the boundary layer at and near separation. In the present work, the condition that $\partial u/\partial y = 0$ at $y = 0$ for all x in the approximating sequence, equation (6), was found to be critical for accurate prediction of the separation point. But this same condition seems to cause inaccurate calculation of $\tau_w(x)$ along the surface. Additionally, the present method does not exhibit a singularity at separation, that is, $d\tau_w/dx \to -\infty$, (Brown and Stewartson [17]) and it is felt that this is also part of the reason for the lack of accuracy in shear stress prediction. As an example of this lack of accuracy, the skin friction coefficient along the surface for the linearly retarded flow, $u_s(x) = u_o(1-X)$, was calculated using the present method. This is shown as a solid line in Fig. 3 along with the finite difference results of Smith and Clutter, [12], the dashed curve. As is evident, the predicted $C_f/2$ by the method is poor in spite of the fact that it predicts the separation point within 0.4% (see Table 1). Also apparent is the more gradual approach of C_f to zero than in the finite difference solution of Smith and Clutter [12]. The explanation for this behavior has been suggested above. If, on the other hand, one uses as the velocity profile, $u(x,y) = u_s y/\delta$, as did Zien [4], in place of our separation profile, equation (6), the predicted $C_f(x)$ are quite good except near separation, but separation itself is predicted at $x_s = 0.142$ instead of at the accepted location $x_s = 0.120$, an error of 18%.

CONCLUDING REMARKS

A form of a modified, or double integral technique has been developed for the specific task of accurately predicting the separation point in laminar boundary layer flows. Agreement is seen to be very good when comparing predicted results with accepted separation point locations for sixteen different free stream velocities.

The concept of the effective velocity profile being used by the double integral method is also established. Comparison of this effective velocity profile with experimental data just upstream of separation and with a series solution result at separation indicated good agreement between them. More generally, it was argued that this effective velocity profile across the boundary layer is much of the explanation for the ability of the double integral method to employ very low order approximating sequences, such as even a simple linear profile, and yet have very accurate predictive capabilities in a wide range of problems in heat and mass transfer.

Also demonstrated was the flexibility of the double integral method. The relatively low order approximating sequences, accompanied by accurate derivatives of the dependent variable found without directly differentiating the original approximating sequences, allows the use of sequences specially designed for a certain type of problem and also the possibility of a simple, yet accurate, analytical solution. This flexibility, which was used in the present work, should also be able to be used to advantage in other problems as well, like aerodynamic heating of an insulated surface with flow near separation.

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